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Gravitational Global Monopoles with Horizons

Dieter Maison

Max-Planck-Institut für Physik
— Werner Heisenberg Institut —
Föhringer Ring 6
80805 Munich (Fed. Rep. Germany)

Abstract

We give arguments for the existence of "radial excitations" of gravitational global monopoles with any number of zeros of the Higgs field and present numerical results for solutions with up to two zeros. All these solutions possess a de Sitter like cosmological horizon, outside of which they become singular. In addition we study corresponding static "hairy" black hole solutions, representing black holes sitting inside a global monopole core. In particular, we determine their existence domains as a function of their horizon radius $r_{\rm h}$.

Like the well-known 't Hooft-Polyakov monopoles, global monopoles are topological solitons with a 'hedgehog' type Higgs field [1]. Whereas the former depend on two mass scales $M_{\rm W}$ and $M_{\rm H}$ – the masses of the Yang-Mills resp. the Higgs field – the latter may be formally obtained by letting $M_{\rm H}$ tend to infinity (keeping $M_{\rm W}$ fixed) and performing a suitable rescaling of the radial coordinate. In this limit the YM field vanishes and the local gauge invariance of the model is lost, only a global SU(2) invariance remaining. This infinite rescaling also sends the mass of the monopole to infinity. When the global monopole is coupled to gravity another mass scale $M_{\rm Pl} = 1/\sqrt{G}$ enters and one obtains a one-parameter family of static global monopoles parametrized by the dimensionless ratio $\eta = M_{\rm W}/M_{\rm Pl}$ [2, 3]. Although the space-times of these solutions are asymptotically flat they have a kind of 'conical' singularity at spatial infinity expressed by a deficit solid angle $\Delta = 4\pi(8\pi\eta^2)$ – a remnant of their infinite mass in flat space. Nevertheless self-gravitating global monopoles have been considered in the literature in connection with 'topological inflation' [4, 5]. This is not unreasonable as the driving mechanism of inflation is an effective cosmological constant provided by the Higgs potential in the region of 'false vacuum' inside the core of the monopole, the asymptotic region far from the monopole playing no role for these considerations. As argued by Linde and Vilenkin static monopoles should undergo inflation when the parameter η exceeds a certain critical value $\eta_{\rm c}$ of order one for which the Schwarzschild radius of the monopole becomes equal to the actual size of the monopole core as measured by $1/M_{\rm W}$. The same type of argument was used before to exclude the existence of static monopoles for $\eta > \eta_{\rm c}$, confirmed by numerical calculations of static solutions [6, 7, 8].

Actually, in the case of global monopoles there is another limit to the existence of static, asymptotically flat solutions obtained when the deficit solid angle Δ becomes equal to 4π, i.e. for $\eta = \sqrt{1/8\pi}$. Nevertheless, as shown by S. Liebling [3], there are still static solutions for larger values of η having a de Sitter like cosmological horizon. Approaching the value $\eta = \sqrt{1/8\pi}$ from above the radius of the cosmological horizon of these solutions tends to infinity. This type of solution exsists up to a maximal value $\eta_{\rm max} = \sqrt{3/8\pi}$ at which the solution is found to bifurcate with the de Sitter solution obtained for vanishing Higgs field. As demonstrated in a recent paper with S. Liebling [10], this bifurcation allows an analytical derivation of $\eta_{\rm max} = \sqrt{3/8\pi}$ related to the existence of a bounded "zero mode". According to standard arguments [11] the latter also implies a change of stability of the de Sitter solution. Whereas the global monopole is stable on its whole domain of existence $0 \leq \eta < \eta_{\rm max}$ the de Sitter solution is unstable in this domain and stable beyond. As was shown in [10] there appear new unstable modes of the de Sitter solution, when η decreases at discrete values η_K accumulating at $\eta = 0$. It does not require a lot of fantasy to surmise that also these points could be points of bifurcation with some new kind of global monopole solutions. Since the corresponding zero modes of the de Sitter solution have Higgs fields with K zeros, these new solutions ought to be "radial excitations" of the global monopole with a corresponding number of zeros. We shall present some numerical evidence that such solutions indeed exist. Some examples are shown in Fig.(1).

As in the case of gravitating 't Hooft-Polyakov monopoles [7, 8] there are also black hole solutions with a "global monopole hair". For the fundamental solution without zero these have been already found in [3]. In the present context we not only describe radial excitations of those, but also determine their existence region in parameter space, consisting of pairs $(\eta, r_{\rm h})$, where $r_{\rm h}$ is the radius of the black hole horizon in Schwarzschild

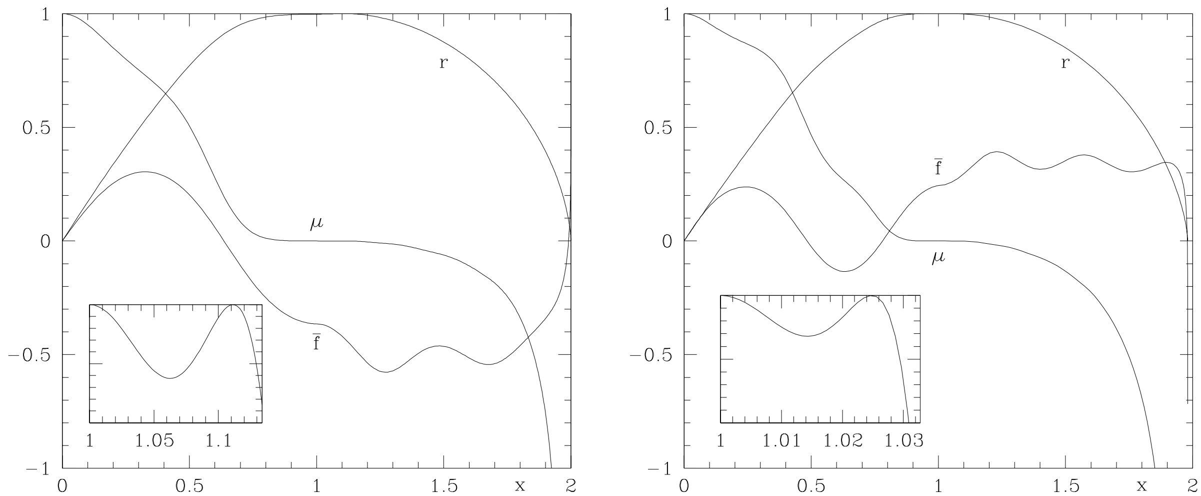

Figure 1: Global monopoles with 1 and 2 zeros corresponding to $\bar{\eta}^2 = 0.3$ and 0.13. The cosmological horizon is situated at $x = 1$. The inserts show enlargements of μ by factors $5 \cdot 10^3$ rsp. 10^7 slightly outside the horizon.

coordinates.

1 Radially excited gravitating global monopoles

Following the notation of [3], we put $\phi^a = f(r)\hat{r}^a$ for the Higgs field and

$$ds^2 = -A^2\mu\, dt^2 + \frac{dr^2}{\mu} + r^2 d\Omega^2 \tag{1}$$

for the spherically symmetric line element in Schwarzschild coordinates. The resulting static field equations are (we prefer to use rationalized units putting $\bar{f} = \sqrt{4\pi} f$, $\bar{\eta} = \sqrt{4\pi}\eta$, $\lambda = 2\pi$ as compared to f and η from [3])

$$\begin{aligned}
\bar{f}' &= \psi && (2)\\
\psi' &= \frac{\bar{f}}{r^2\mu}\Big[2 + \frac{r^2}{2}(\bar{f}^2 - \bar{\eta}^2)\Big] - \psi\Big[\frac{2}{r} + r\psi^2 + \frac{\mu'}{\mu}\Big] && (3)\\
\mu' &= \frac{1-\mu}{r} - r\mu\psi^2 - \frac{2\bar{f}^2}{r} - \frac{r}{4}(\bar{f}^2 - \bar{\eta}^2)^2 && (4)\\
A' &= r\psi^2 A. && (5)
\end{aligned}$$

Solutions with a regular origin obey the boundary conditions

$$\bar{f}(r) = ar + O(r^3), \quad \psi(r) = a + O(r^2), \quad \mu(r) = 1 + O(r^2), \quad A(r) = A_0 + O(r^2) \tag{6}$$

and are uniquely specified by the choice of $a = \psi(0)$ and A_0.

The boundary conditions for black holes at the horizon are

$$\begin{aligned}
\bar{f}(r) &= \bar{f}_{\rm h} + O(r - r_{\rm h}), \quad \psi(r) = \psi_{\rm h} + O(r - r_{\rm h}), && (7)\\
\mu(r) &= \mu'_{\rm h}(r - r_{\rm h}) + O((r - r_{\rm h})^2), \quad A(r) = A_{\rm h} + O((r - r_{\rm h})^2) && (8)
\end{aligned}$$

with

$$\mu'_{\rm h} = \frac{1}{r_{\rm h}}\Big[1-2\bar f_{\rm h}^2-\frac{r_{\rm h}^2}{4}(\bar f_{\rm h}^2-\bar\eta^2)^2\Big]>0 \tag{9}$$

$$\psi_{\rm h} = \frac{\bar f_{\rm h}}{r_{\rm h}^2\mu'_{\rm h}}\Big[2+\frac{r_{\rm h}^2}{2}(\bar f_{\rm h}^2-\bar\eta^2)\Big]\,. \tag{10}$$

Solutions with any of these boundary conditions stay finite for increasing r as long as μ is non-zero. However, generically μ vanishes for some finite value of r. Depending on whether A and ψ stay finite or not at the zero of μ the geometrical significance of such points is different. In the first case the solution has a cosmological horizon, in the latter it has an "equator", i.e. a maximum of r considered as a metrical function (compare eq.(1)). The boundary conditions at a cosmological horizon are the same as at a black hole horizon given above with the only difference that $\mu'_{\rm h}<0$.

Solutions with a regular origin resp. black hole boundary conditions and a cosmological horizon can be obtained by fine-tuning the parameter a resp. $\bar f_{\rm h}$. An important special case is obtained for $a=0$ resp. $\bar f_{\rm h}=0$ yielding the de Sitter resp. Schwarzschild-de Sitter solution with a cosmological constant provided by the Higgs potential for $\bar f\equiv 0$. The de Sitter (dS) solution is given by

$$\mu_{\rm dS}(r)=1-\frac{r^2}{r_{\rm c}^2}\quad\text{with}\quad r_{\rm c}=\frac{2\sqrt3}{\bar\eta^2}\,, \tag{11}$$

whereas the Schwarzschild-de Sitter (SdS) solution reads

$$\mu_{\rm SdS}(r)=1-\frac{r^2}{r_{\rm c}^2}-\frac{r_{\rm h}}{r}\Big(1-\frac{r_{\rm h}^2}{r_{\rm c}^2}\Big)\,. \tag{12}$$

Obviously the SdS solution goes into the dS solution for $r_{\rm h}\to 0$. The cosmological horizon of the SdS solution turns out to be located at

$$r_+=-\frac{r_{\rm h}}{2}+\sqrt{r_{\rm c}^2-\frac{3}{4}r_{\rm h}^2} \tag{13}$$

As was shown in [10] the de Sitter solution has bounded "zero modes" φ_K solving the linearized eqs.(2) in the dS background for the discrete values

$$\bar\eta_K^2=\frac{3}{(K+2)(2K+1)}\qquad\text{for}\quad K=0,1,2,.. \tag{14}$$

At the largest one $\bar\eta_0^2=3/2$ the de Sitter solution bifurcates with the global monopole solutions possessing a themselves a cosmological horizon for $\bar\eta^2>1/2$ [3]. The corresponding zero mode is given by $\varphi_0=\partial\bar f/\partial a$, where a is the parameter characterizing the solution at $r=0$ (compare eq.(6)). As was already mentioned above we may look for "radially excited" global monopoles bifurcating with the de Sitter solution at the other zero modes φ_K for $K>0$. Since the functions φ_K have K zeros, we have to look for solutions of eqs.(2) with $\bar f$'s with the same property and and a cosmological horizon. The results of a numerical investigation for the cases $K=1,2$ are summarized in Fig.(2), in which the values of the fine-tuning parameter a are plotted as a function of $\bar\eta^2$. The graphs of the functions $a(\bar\eta^2)$ look very similar to the one for the case $K=0$ given in [10] and approach the latter for $\bar\eta\to 0$.

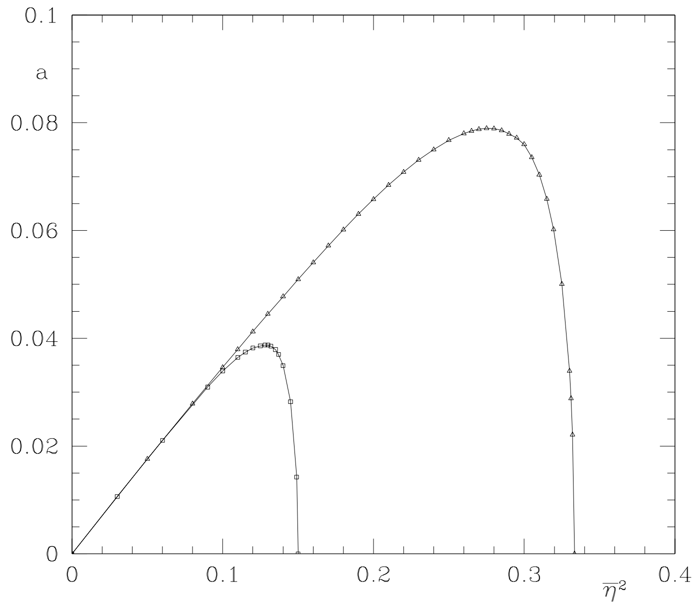

Figure 2: $a(\bar{\eta}^2)$ for the first and second radially excited global monopoles

There is an important difference in the global behaviour between the fundamental monopole and the radially excited solutions. Whereas the first ones extend to arbitrarily large values of r with $\bar{f} \to \bar{\eta}$ and $\mu \to 1 - 2\bar{\eta}^2$, the latter ones have an "equator" (maximum of r) just outside their cosmological horizon and then run back to $r = 0$, where they become singular. The behaviour beyond the equator is quite similar to the one of "hairy" black holes inside their horizon [9]. Obviously μ cannot tend to $1 - 2\bar{\eta}^2$ for $\bar{\eta}^2 < 1/2$ without becoming positive again. This, however, could only be achieved with finite ψ by further fine-tuning, for which there is no parameter left for given $\bar{\eta}$. One might try to also fine-tune $\bar{\eta}$ in order find such solutions, but then one would still need another parameter to suppress the divergent mode of the Higgs field for $r \to \infty$ to enforce $\bar{f} \to \bar{\eta}$.

Since the Schwarzschild coordinates become singular at the equator, where r is stationary one has to use a different radial coordinate. A convenient choice is to take the geodesic distance s in the radial direction and determine $r(s)$ solving

$$\frac{d}{ds} r = \sqrt{|\mu|}\,. \tag{15}$$

Fig.(1) shows the functions $\bar{f}(x)$, $\mu(x)$ and $r(x)$ for particular values of $\bar{\eta}$, where the coordinate x is equal to s up to a normalisation factor chosen such that the cosmological horizon where μ vanishes is situated at $s = 1$. The equator at a second zero of μ is very close to, but slightly outside the horizon. Since μ stays very small between its two zeros we have inserted enlargements in Fig.(1) in order to make this behaviour visible.

2 Black holes with global monopole hair

Next we turn to black holes sitting inside global monopoles. Such solutions have been described already by S. Liebling [3]. According to his results there is a 2-parameter family of such solutions, parametrized by the radius r_{h} of their bh-horizon and $\bar{\eta}$. Encouraged by the results of the previous section, we also look for black holes sitting inside radially excited global monopoles. Indeed, such solutions are readily found numerically solving

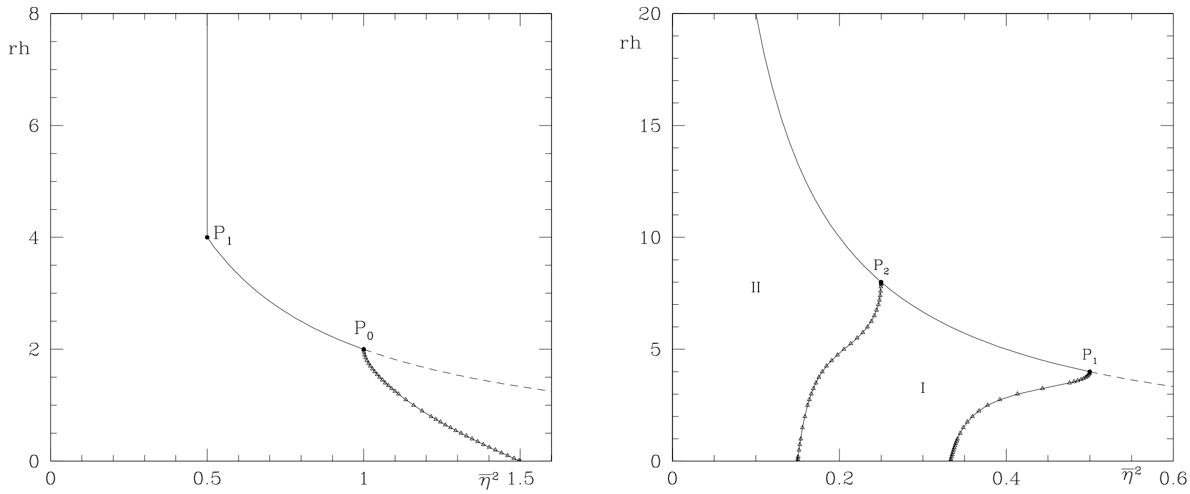

Figure 3: Existence domains for black holes. The first plot refers to the fundamental solution, the second one to the first and second radial excitations. The first radial excitation exists in the union of regions I and II, whereas the second one exists only in region II.

eqs.(2) with bh boundary conditions eq.(7) and analogous ones for a cosmological horizon at some $r_{\rm c} > r_{\rm h}$.

In view of the experience with hairy black holes studied in the literature [8], we expect a limited domain of existence in the $\bar{\eta}-r_{\rm h}$-plane for such solutions. The numerical analysis shows that there is a qualitative difference between the fundamental solutions (without zeros of the Higgs field) and the radially excited ones. Let us first discuss the fundamental solutions.

As shown in Fig.(3) there three regions in the $\bar{\eta}-r_{\rm h}$-plane. For $0 < \bar{\eta}^2 < 1/2$ black holes with arbitrarily large radius $r_{\rm h}$ seem to exist. In the interval $1/2 < \bar{\eta}^2 < 1$ the $r_{\rm h}$ values are bounded from above by the curve $r_{\rm h} = 2/\bar{\eta}^2$. Approaching this curve from below the solutions bifurcate with the SdS solution (i.e. $\bar{f} \to 0$), such that at the same time $r_{\rm h}$ tends to $r_{\rm c}$. Comparing with eq.(13) this yields the relation $r_{\rm h} = r_{\rm c}/\sqrt{3} = 2/\bar{\eta}^2$.

In the interval $1 < \bar{\eta}^2 < 3/2$ the existence domain is bounded by some curve joining the points $(1, 2)$ and $(\sqrt{3/2}, 0)$ in the $\bar{\eta} - r_{\rm h}$-plane. Approaching this curve the solution again bifurcates with the SdS solution, but this time with $r_{\rm h} < r_+$. As in the case of the regular solution the bifurcation points are determined by the requirement that a bounded zero mode of the SdS solution exists. The equation to be solved for $\varphi = r\delta\bar{f}$ is (using rescaled radial variables $x = r/r_{\rm c}$ and $x_{\rm h} = r_{\rm h}/r_{\rm c}$)

$$\frac{d}{dx}(\mu_{\rm SdS}\frac{d}{dx}\varphi) = \Big(\frac{2}{x^2} - 2 - \frac{6}{\bar{\eta}^2} - \frac{x_{\rm h}^3 - x_{\rm h}}{x^3}\Big)\varphi \qquad (16)$$

with bh boundary conditions at $x = x_{\rm h}$ and $\mu_{\rm SdS}$ as given by eq.(12). The requirement for φ to stay bounded for $x \to r_+/rc$ then determines the boundary curve $r_{\rm h}(\bar{\eta})$ shown in Fig.(3) joining the points P_0 and Q_0.

The situation is quite similar for the radially excited solutions, the only difference being the absence of the unbounded piece of the domain. The existence domains for solutions with one and two zeros of $\bar{f}$ are shown in Fig.(3). The intersection points $P_i, i = 0, 1, 2$, etc. of the respective boundary curves can be determined analytically as follows.

From what was said above they are determined by the condition that $r_{\rm h} \to r_+$ as one approaches them along the curve determined by the zero mode condition. In order to study this limit we assume $x_{\rm h} = 1/\sqrt{3} - \epsilon$ with $\epsilon << 1$ and introduce a rescaled variable y defined by $x = \frac{1}{\sqrt{3}} + \epsilon y$. Keeping only the leading terms as $\epsilon \to 0$ we obtain from eq.(16)

$$\frac{d}{dy}\Big((1-y^2)\frac{d}{dy}\varphi\Big) = 2(1-\frac{1}{\bar{\eta}^2})\varphi\,. \tag{17}$$

When x varies from $x_{\rm h}$ to r_+/rc the variable y runs from -1 to $+1$. The bounded solutions on this interval are the Legendre polynomials $P_K(y)$ obtained for

$$\bar{\eta}_K^2 = \frac{2}{K(K+1)+2} \qquad K = 0, 1, ... \tag{18}$$

The first three values are $\bar{\eta}^2 = 1, 1/2, 1/4$ with corresponding values $r_{\rm h} = 2, 4, 8$ matching exactly the points P_0, P_1 and P_2 of Fig.(3).

3 Summary

In section 1 we put forward some arguments for the existence of radial excitations of the static gravitational global monopoles possessing a de Sitter like cosmological horizon studied in [3, 10]. We present some numerical evidence for the existence of solutions with up to two zeros of the Higgs field. In section 2 we study corresponding static hairy black hole solutions, representing black holes sitting inside a global monopole core. In particular, we determine their existence domains as a function of their horizon radius $r_{\rm h}$.

In a forthcoming publication we shall consider generalisations of these results to gravitational monopoles with a dynamical YM field.

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